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CONSTRUCTING A PERPENDICULAR BISECTOR USING COMPASS AND STRAIGHTEDGE: A FUNDAMENTAL GEOMETRIC METHOD

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ABSTRACT

This article investigates one of the most elementary yet fundamental problems in classical Euclidean geometry: constructing the perpendicular bisector of a line segment using only a compass and straightedge. The study outlines the theoretical background, presents a step-by-step construction method, and discusses its applications in both historical and modern mathematical contexts. The simplicity and power of this method reveal its pedagogical value in developing geometric intuition and precision.

KEYWORDS: Compass and straightedge, perpendicular bisector, geometric construction, Euclidean geometry, basic constructions.

INTRODUCTION

Geometric construction using only a compass and straightedge is a classical discipline that lies at the heart of Euclidean geometry. Dating back to ancient Greek mathematicians such as Euclid, Pythagoras, and Apollonius, these constructions were developed as rigorous tools for exploring the properties of space without the aid of measurements or coordinates. One of the foundational problems in this discipline is the construction of a perpendicular bisector—a line that not only divides a segment into two equal parts but also forms a right angle with it. This simple yet powerful construction forms the basis for more advanced geometric concepts such as the construction of triangle altitudes, circumcenters, and symmetry axes.

In classical texts like The Elements by Euclid, constructions were built on a system of axioms and postulates, where each new method relied solely on previously proven constructions and logical reasoning. The perpendicular bisector problem is typically derived from Postulates 1–3, involving drawing lines and circles between two points. It exemplifies how complex structures in geometry emerge from minimal tools and foundational logic.

Beyond its historical significance, this construction remains a central pedagogical tool in modern mathematics education. It enhances students' understanding of spatial reasoning, symmetry, and congruence, while also introducing them to the logical structure of mathematical proofs. In today's technological world, where software often replaces manual construction, learning the compass and straightedge method helps reinforce geometric intuition and fosters a deeper conceptual understanding of Euclidean principles.

NEXT SCIENTISTS CONFERENCES

This paper explores the construction of the perpendicular bisector using compass and straightedge, provides a step-by-step geometric algorithm, and discusses its mathematical justification and applications in both historical and contemporary settings.

METHODS

The geometric task is to construct the perpendicular bisector of a given line segment ¯AB, using only two classical tools: a compass and an unmarked straightedge. This problem is a fundamental component of classical construction and is closely aligned with Euclid's Postulates 1, 2, and 3: the ability to draw straight lines between points, extend a line indefinitely, and draw a circle with any center and radius.

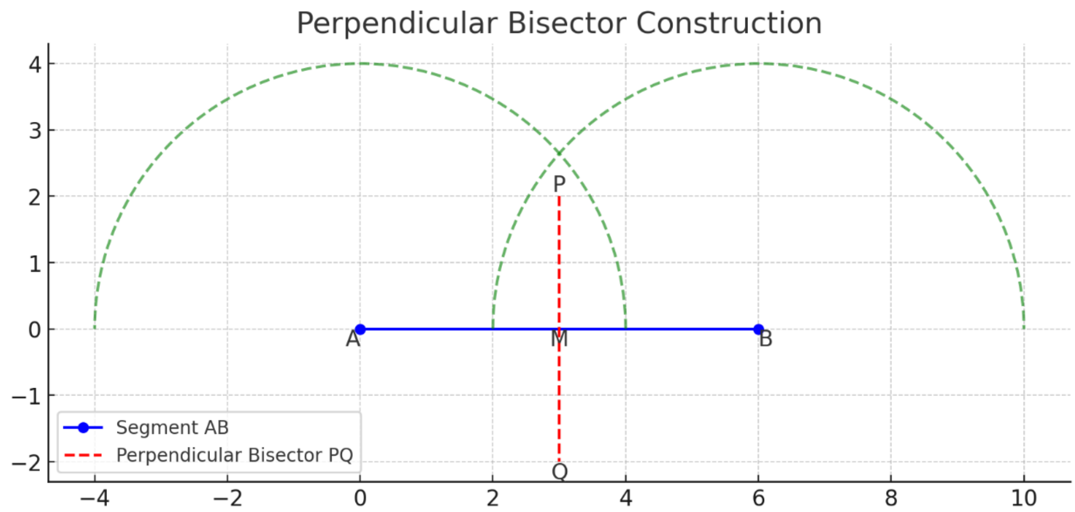
Let A and B be the endpoints of a line segment $\overline{AB}$. The goal is to construct a line ℓ that is perpendicular to $\overline{AB}$ and passes through its midpoint M.

Step 1: Place the compass point on endpoint AAA, and draw an arc of radius $r > \frac{1}{2} \cdot AB$. This arc should intersect the plane both above and below the segment.

Step 2: Without changing the compass radius, repeat the same procedure from endpoint B, creating two intersection points with the previous arcs—denote them as P (above the segment) and Q (below the segment).

Step 3: Use the straightedge to draw the line segment $\overline{PQ}$. This line intersects $\overline{AB}$ at its midpoint M, and is perpendicular to it by symmetry.

Visual explanation: The arcs from points A and B define a pair of congruent triangles: $\triangle PAB \cong \triangle QAB$, by SSS (Side-Side-Side) congruence. The line $\overline{PQ}$, connecting the intersection points of these arcs, thus becomes the locus of points equidistant from A and B, satisfying the definition of a perpendicular bisector.



Here is a visual representation of the perpendicular bisector construction. The segment $\overline{AB}$ is bisected by the red dashed $line$ $\overline{PQ}$, which passes through the midpoint M and forms a 90° angle. The green arcs illustrate the equal-radius constructions from points A and B, confirming the use of symmetry and congruence.

Let $A(x_1, y_1)$ and $B(x_2, y_2)$. The midpoint M of $\overline{AB}$ is calculated as:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The slope m of segment $\overline{AB}$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Then, the slope $m \perp$ of the perpendicular bisector is:

$$m_\perp = -\frac{1}{m}$$

Thus, the perpendicular bisector has the equation:

$$y - y_M = -\frac{1}{m}(x - x_M)$$

This analytical confirmation supports the geometric method, ensuring the constructed line is both perpendicular to $\overline{AB}$and passes through its midpoint.

Postulate 1: A straight line may be drawn from any point to any point.
Postulate 2: A finite straight line may be extended continuously in a straight line.
Postulate 3: A circle may be drawn with any center and any radius.

This construction illustrates how these foundational postulates allow the generation of perpendicular and symmetric figures without the use of measurement, upholding the axiomatic integrity of classical geometry.

RESULTS

The construction yields a line $\overline{PQ}$ that intersects the segment $\overline{AB}$ exactly at its midpoint M, satisfying the condition $AM = MB$. Furthermore, the angle formed between $\overline{PQ}$ and $\overline{AB}$ is 90°, verifying that $\overline{PQ}$ is the perpendicular bisector of $\overline{AB}$. This conclusion is substantiated by the congruence of the radii used to draw intersecting arcs from points A and B, which create two intersection points-P and Q—that are equidistant from A and B.

Notably, the entire construction is achieved without numerical measurement; it is based solely on fundamental geometric principles such as congruence, symmetry, and equidistance. The resulting configuration inherently satisfies both the midpoint and perpendicularity properties due to the reflective symmetry across the line $\overline{PQ}$. This not only confirms the theoretical accuracy of the construction but also demonstrates its pedagogical value in illustrating key concepts such as bisectors, symmetry axes, and elementary geometric proofs.

DISCUSSION

This classical geometric construction highlights the elegance, logical rigor, and visual symmetry inherent in Euclidean geometry. The perpendicular bisector, constructed solely through compass-and-straightedge techniques, is more than a basic exercise-it is a foundational tool with far-reaching implications in both pure and applied mathematics.

From a pedagogical standpoint, this construction is invaluable. It introduces learners to: geometric reasoning through symmetry and congruence, the axiomatic method, where complex outcomes are derived from simple, self-evident truths, the process of formal proof, as students can justify each step logically without relying on measurement.

Moreover, the perpendicular bisector is a gateway construction to more advanced geometric concepts:

-It is essential in the construction of triangle medians, altitudes, and angle bisectors.

-It plays a critical role in the creation of circumcircles, as the perpendicular bisectors of a triangle's sides intersect at the circumcenter-a point equidistant from all vertices.

It appears in coordinate geometry, where algebraic techniques validate geometric properties, and in transformational geometry, where reflections over a line (such as a perpendicular bisector) form the basis of symmetry operations.

In a broader mathematical context, this construction embodies the philosophical clarity of Euclidean methods. By relying purely on relative position and equality, rather than numerical data, it fosters abstract thinking and cultivates an appreciation for the universality of geometric truths-applicable in architecture, engineering, design, and computer graphics.

Finally, when this construction is placed alongside non-Euclidean or projective interpretations (e.g., Lobachevskian geometry), it provides a comparative platform to explore how fundamental assumptions shape mathematical systems. In such comparative studies, Euclidean constructions like this one serve as a reference point from which other geometries diverge.

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